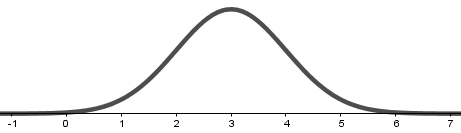
Calculating Probabilities with the Normal distribution

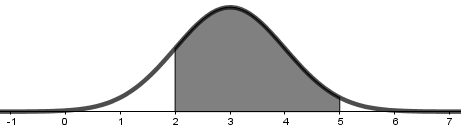
# Probabilities from the Normal distribution

Once a Normal distribution has been declared for a random variable we are able to use this distribution to find probabilities associated with the random variable. Suppose that is a random variable and we are to model it as . Then the probability that is between two values, and will be given by the area under the bell curve between and . We call this .

For example, if .

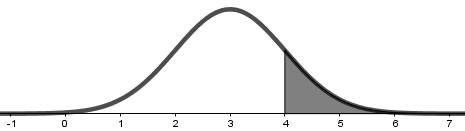


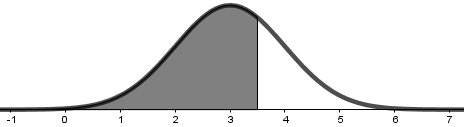
Then the probability that is between and is the area under the bell curve between and .



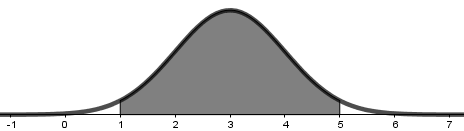
In this example, it turns out that , which is 82%.

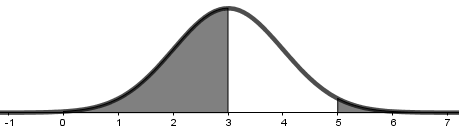
It’s also possible to ask for one-sided probabilities, such as and





We can also ask for slightly more complicated probabilities, like and .

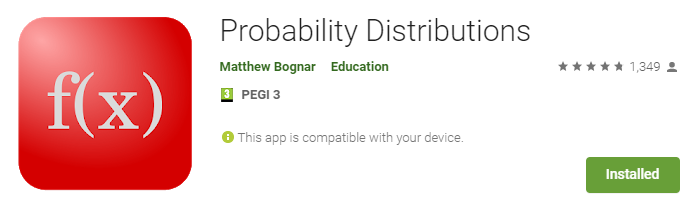




How can we actually calculate these probabilities? Unfortunately, we can’t use any simple techniques to calculate the area under a bell curve, we must resort to numerical approximation techniques. Luckily, in this modern age, there are many options for calculating these probabilities.

# Method 1 Mobile Phone App

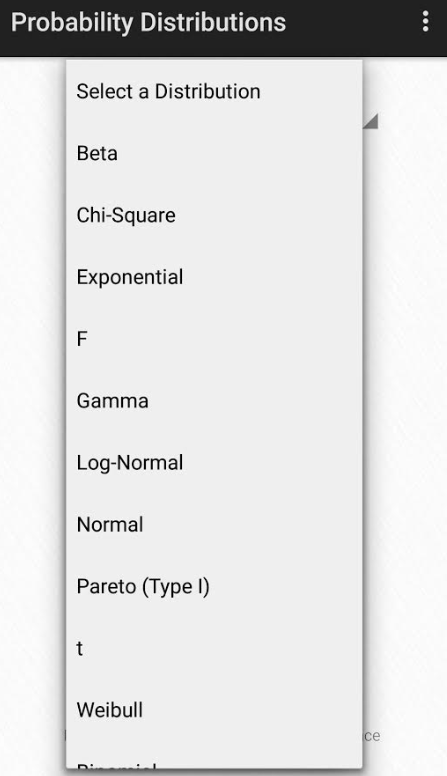
There are a multitude of mobile phone apps that can calculate Normal probabilities. One such app is Matthew Bognar’s free *Probability Distributions* app, available on Google Play and the iPhone App Store.



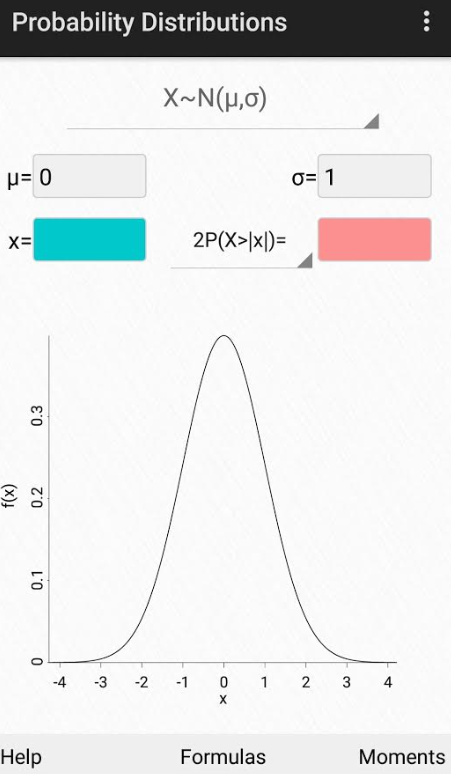
Let’s look at how to find if .

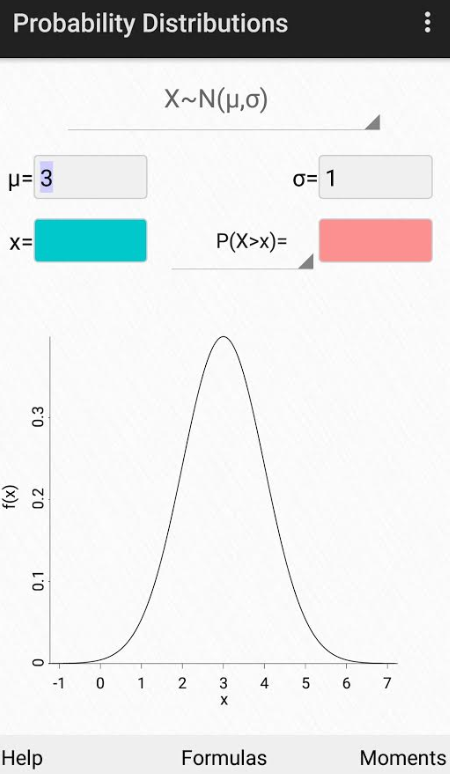
First open the app and select the Normal distribution.





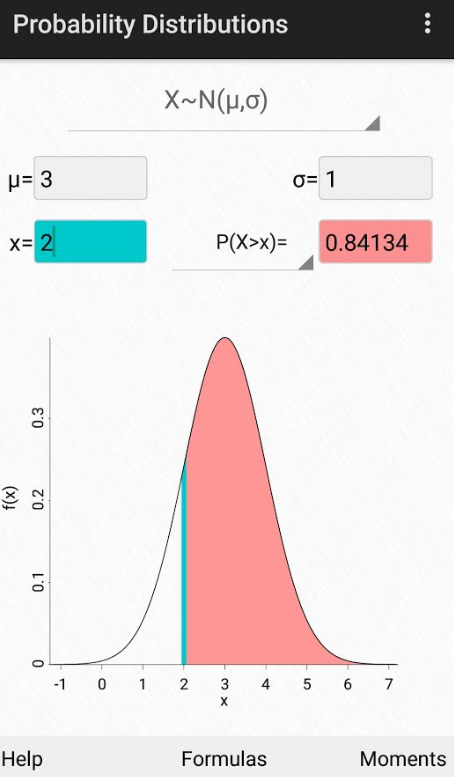
Choose the appropriate value for and . Note that this app wants to know only, not which is the value used in the declaration of the Normal distribution. For example, if then . In this case, with , and also.

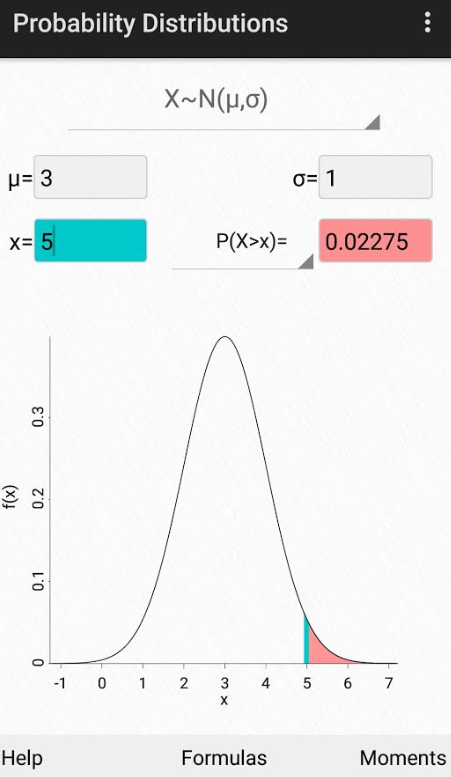




Make sure that P(X>x) is selected. This means that the app will calculate when you place in the blue box on the left.

Since the app can only provide , we need to slightly manipulate the probability we are after. We note that. These two probabilities can now be found with the app.





Using these value, we are able to complete the calculation. .

# Method 2 Computer Software

There any many specific mathematical and statistical software packages such as R, SPSS, Minitab, Matlab, GeoGebra, etc, that can provide probability calculations for the Normal distribution. Some are free, some will be available under licences to your university, but others will require a purchase.

An alternative to all of these is Microsoft Excel, which comes with a special function NORMAL.DIST. To use this function, simply type

=NORMAL.DIST(

into any cell of an Excel worksheet. If you’re using a recent version of Excel, it should prompt you to supply four inputs; x, mean, standard\_dev, cumulative.



* x is the value of that you are interested in
* mean is the value of the mean of ,
* standard\_dev is the value of the standard deviation of , ; note this is *not* the variance,
* cumulative needs to be set to TRUE, this produces a cumulative probability from the left tail

Here are some examples.

NORMAL.DIST(1,2,3,TRUE) will produce where

NORMAL.DIST(3,1,2,TRUE) will produce where

NORMAL.DIST(-2,0,1,TRUE) will produce where

NORMAL.DIST(2,-1,0.5,TRUE) will produce where

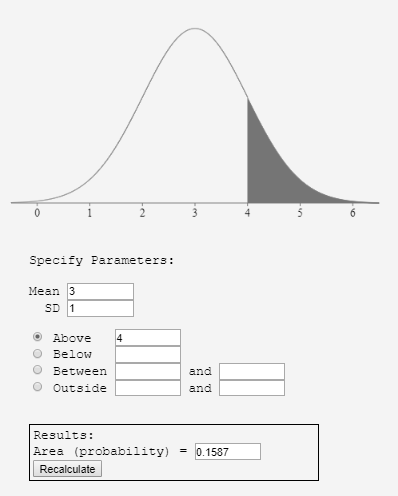
In order to calculate a more complicated probability. For example, if , then

Alternatively,

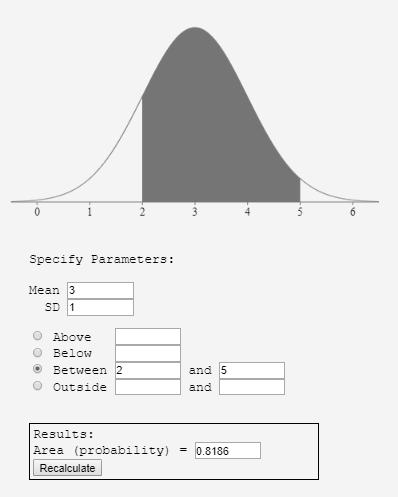
# Method 3 Ask the Internet

A quick little Google of the search term *normal probability calculator* gives a multitude of online calculators.

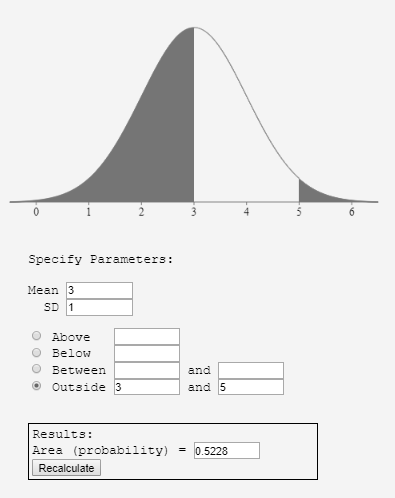
For example, the *OnlineStatBook* Normal probability calculator ([link](http://onlinestatbook.com/2/calculators/normal_dist.html)) is very flexible; it allows you to calculate Normal probabilities very directly.



Here we calculate if .



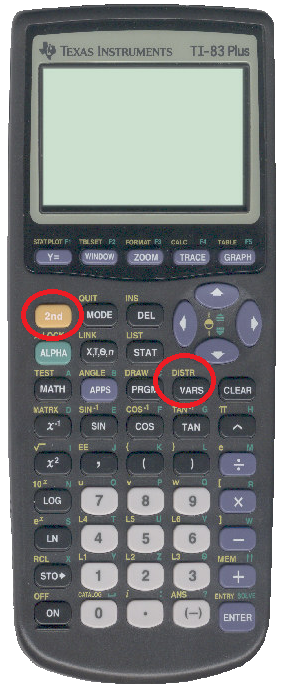
Here we calculate if .



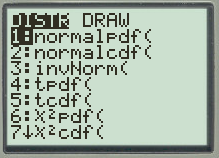
And here we calculate if .

# Method 4 Graphics Calculator

If you have a graphics calculator, then you might have access to special functions for probability distributions. For example, the *Texas Instruments TI-83 Plus* calculator has a DISTRIBUTIONS menu that you can access by pressing 2nd + DIST.



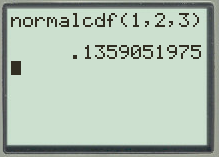
The first three functions are based on the Normal distribution.



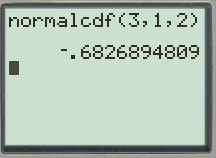
For calculating probabilities, we want to use normalcdf (which stands for Normal cumulative distribution function). The format required is normalcdf(,,) and this will calculate where . It’s similar in set-up to the Microsoft Excel function.

Here are some examples.

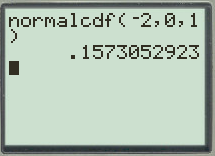
normalcdf(1,2,3) will produce where



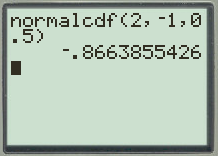
normalcdf(3,1,2) will produce where



normalcdf(-2,0,1) will produce where



normalcdf(2,-1,0.5) will produce where



For any probabilities not in the format, you will need to make similar adjustments as mentioned previously.

# Method 5 Statistical Tables

This method is not as useful as it once was before the onset of modern computing. However, it is still important to know how to use statistical tables to calculate Normal probabilities.

Firstly, it is obviously not possible to create lists of Normal probabilities for all combinations of and and so most tables only list probabilities for what’s called the *standard Normal distribution*, . We often use the letter to represent the standard Normal random variable, such that .

In order to calculate probabilities with respect to generic Normal random variable, , we use the transformation

This is often described as *normalising* the random variable to . A nice property of the Normal distribution is that all probabilities will be maintained with respect to the variables’ individual means and standard deviations.

For example, if ( and ) and ( and ), then

The probability each variable is less than its

The probability each variable is more than 1 above its

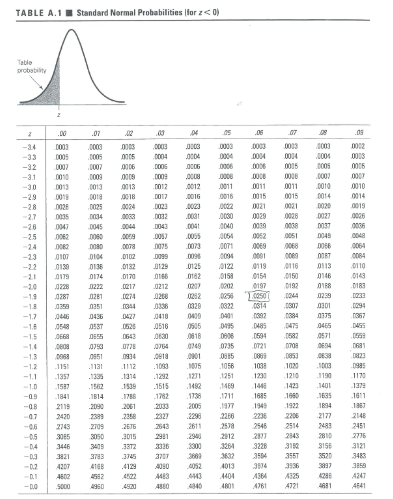
The probability each variable is within of its

Each of these equalities can be achieved with the transformation

The second step is to look up these probabilities in terms of on a statistical table. These statistical tables, unfortunately, come in many forms and so it’s important to check exactly what they will do.

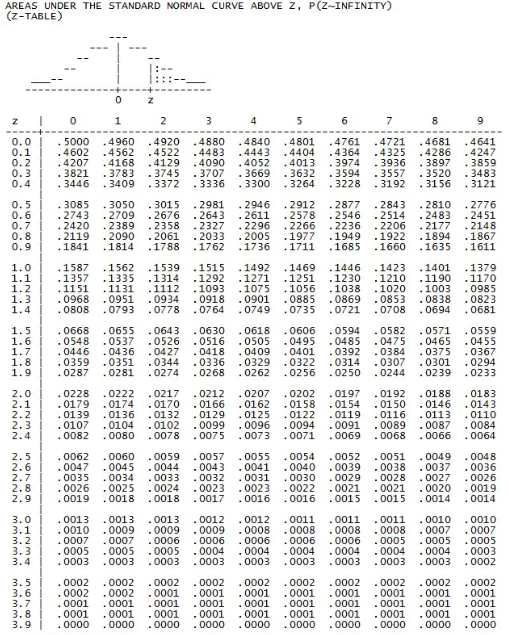
Some statistical tables only give left-tail probabilities less than 50%.

Table



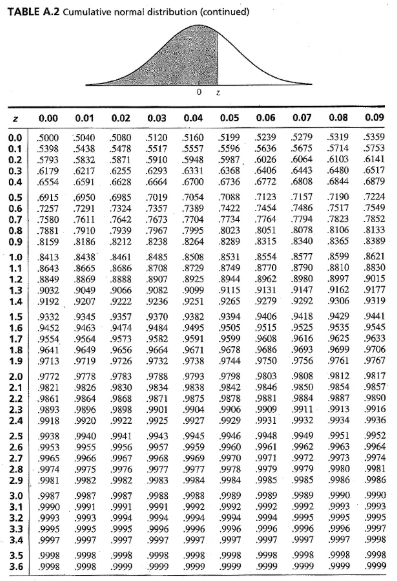
Some statistical tables only give right-tail probabilities less than 50%.

Table



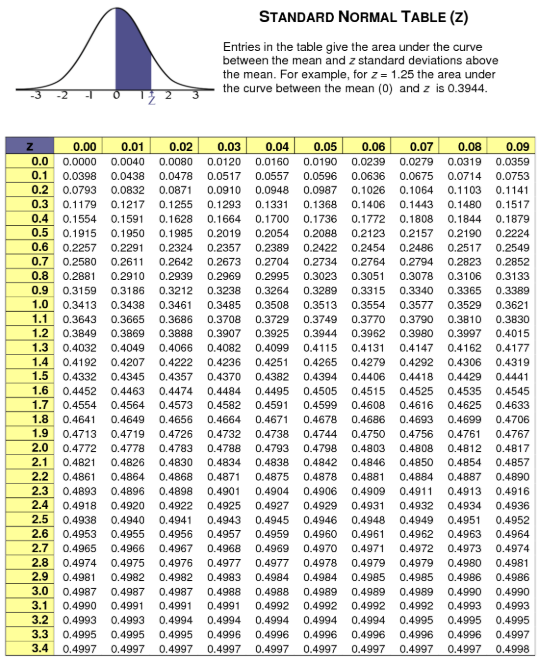
Some statistical tables only give left-tail probabilities *greater* than 50%.

Table



Some statistical tables only give probabilities from to a positive value of .

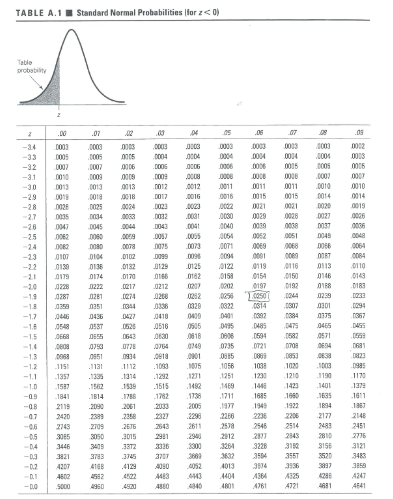
Table



Either way, you will simply need to manipulate the probabilities you seek into the format required of the table you are using. You can do this by either using symmetry, or the fact that complementary values must add to .

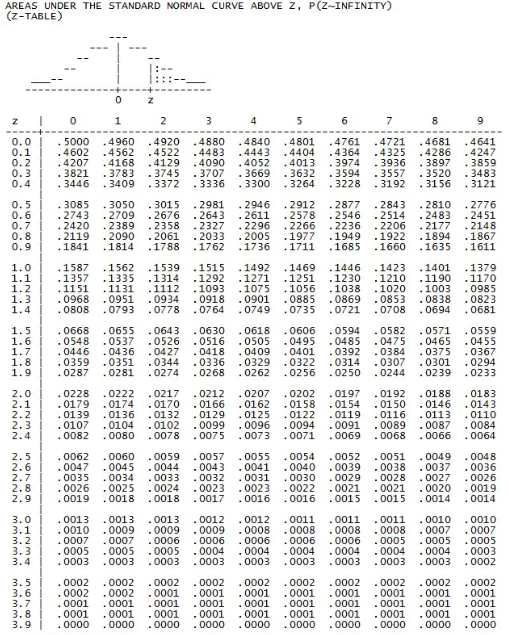
For example, suppose and we wish to calculate . We first normalise a random variable.

If we were using Table 1, which requires left-tail probabilities less than 50%,



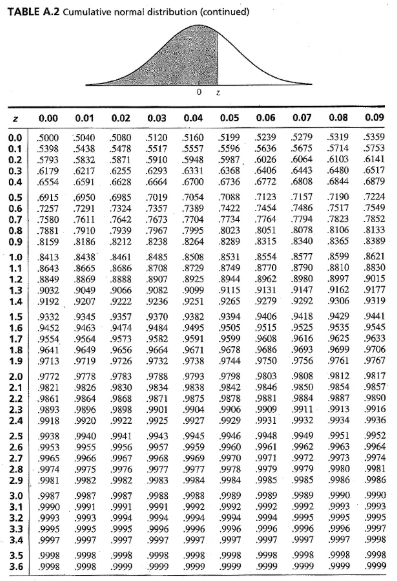
We would rearrange to the following.

If we were using Table 2, which requires right-tail probabilities less than 50%.



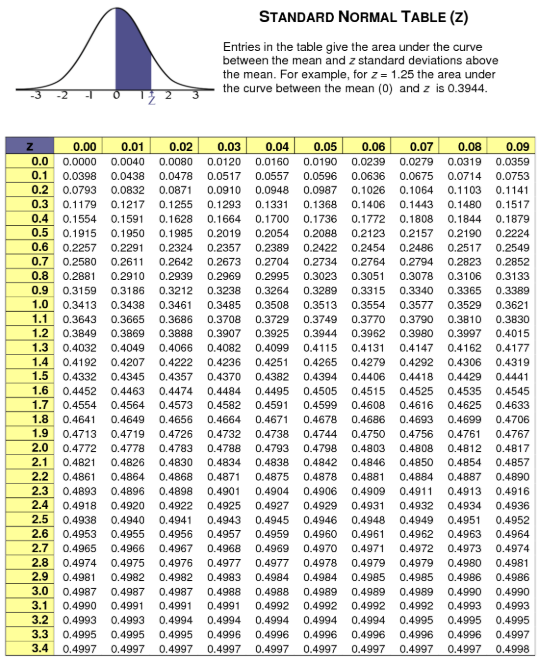
We would rearrange to the following.

If we were using Table 3, which requires left-tail probabilities greater than 50%.



We would rearrange to the following.

If we were using Table 4, which requires probabilities from to a positive value of .



We would rearrange to the following.

Activity

Using the method of your choice, calculate the following probabilities.

1. where
2. where
3. where
4. where
5. where
6. where
7. where
8. where
9. where
10. where
11. where
12. where
13. where
14. where
15. where
16. where